## Math Department Curriculum Guide

Calculus – AP

#### **Course Description**

AP Calculus AB focuses on students' understanding of calculus concepts and provide experience with methods and applications. Through the use of big ideas of calculus (e.g., modeling change, approximation and limits, and analysis of functions), each course becomes a cohesive whole, rather than a collection of unrelated topics. Both courses require students to use definitions and theorems to build arguments and justify conclusions. The courses feature a multi-representational approach to calculus, with concepts, results, and problems expressed graphically, numerically, analytically, and verbally. Exploring connections among these representations builds understanding of how calculus applies limits to develop important ideas, definitions, formulas, and theorems. A sustained emphasis on clear communication of methods, reasoning, justifications, and conclusions is essential. Teachers and students should regularly use technology to reinforce relationships among functions, to confirm written work, to implement experimentation, and to assist in interpreting results. Any student who enrolls in an AP course is required to take the AP exam in May of the school year.

Course Content	Mathematical Practices
<ul> <li>The AP Calculus AB course is organized into eight units arrananged in a logical sequence.</li> <li>Unit 1: Limits and Continuity</li> <li>Unit 2: Differentiation: Definition and Properties</li> <li>Unit 3: Differentiation: Composite and Implicit</li> <li>Unit 4: Contextual Applications</li> <li>Unit 5: Analytical Applications</li> <li>Unit 6: Integration and Accumlation of Change</li> <li>Unit 7: Differential Equations</li> <li>Unit 8: Applications of Integration</li> </ul>	<ul> <li>Implementing Mathematical Processes: Determine expressions and values using mathematical procedures and rules.</li> <li>Connecting Representations: Translate mathematical information from a single representation or across multiple representations.</li> <li>Justification: Justify reasoning and solutions.</li> <li>Communication and Notation: Use correct notation, language, and mathematical conventions to communicate results or solutions.</li> </ul>

### **Calculator Skills**

It is important that students have access to and learn to use an assessment-approved graphing calculator. The graphing calculators skills essential to succes in thai course include:

- Basic operations with fractions and exponents
- Convert between decimals and fractions
- Enter equations in y =
- Manipulate the window
- Manipulate the table
- ➤ Graphing functions
- > Analyzing functions in three ways (table, graph, and equation)
- ➤ Finding point of intersection
- Finding maximums and minimums
- ➤ Finding zeros
- Calculate derivatives
- Calculate integrals



# Subject: Calculus – AP

Units	Content
<b>Unit 1 - Limits and Continuity</b> Term 1 September	<ul> <li>Rates of change average and instantaneous</li> <li>Limits at a point - one and two-sided limits</li> <li>Limits involving infinity - properties, end behavior</li> <li>Continuity - at a point, types of discontinuity</li> <li>Intermediate Value Theorem</li> <li>Rates of change - slope of a curve, normal line, speed</li> </ul>
<b>Unit 2 - Derivatives</b> Term 1 October and November	<ul> <li>Derivative of a function</li> <li>Relationships between the graphs of f and f"</li> <li>Graph the derivative from data</li> <li>How the derivative fails to exist</li> <li>Local linearity</li> <li>Calculate the derivative on a calculator</li> <li>Differentiability implies continuity</li> <li>Rules for differentiation</li> <li>Higher order derivatives</li> <li>Velocity and other rates of change</li> <li>Derivative of trigonometric functions</li> <li>Chain rule</li> <li>Implicit differentiation</li> <li>Derivative of inverse trigonometric functions</li> <li>Derivative of exponential and logarithmic functions</li> </ul>
<b>Unit 3 - Application of Derivatives</b> Term 2 November and December	<ul> <li>Extreme values of functions - absolute extrema, relative extrema</li> <li>Extreme Value Theorem</li> <li>Definition of critical point</li> <li>Mean Value Theorem</li> <li>Rolle's Theorem</li> <li>Analysis of graphs using the first and second derivative test</li> <li>Concavity and points of inflection</li> <li>Optimization problems</li> <li>Linearization - tangent line approximations and differentials</li> <li>Related rate word problems</li> </ul>



Units	Content
<b>Unit 4 - The Definite Integral</b> Term 2 January and February	<ul> <li>Estimating with finite sums</li> <li>Calculate distance traveled</li> <li>RAM</li> <li>Trapezoidal sums</li> <li>Definite integrals and area</li> <li>Properties of definite integrals</li> <li>Average value theorem</li> <li>Mean Value Theorem for definite integrals</li> <li>Fundamental Theorem of Calculus</li> <li>Analyize anti-derivatives graphically</li> </ul>
Unit 5 - Differential Equations and Mathematical Modeling Term 3 February and March	<ul> <li>Slope fields</li> <li>Evaluating definite integrals</li> <li>Indefinite integrals</li> <li>Anti-differentiation by substitution</li> <li>Solving separable differential equations</li> <li>Growth and decay problems</li> <li>General differential equations</li> </ul>
Unit 6 - Application of Definite Integrals Term 3 March	<ul> <li>Integral as net change</li> <li>Particle motion</li> <li>Consumption over time</li> <li>Net change from data</li> <li>Area between a curve and an axis (integrating with respect to x and y)</li> <li>Area between intersecting curves (integrating with respect to x and y)</li> <li>Volume of solids with known cross sections</li> <li>Volumes of solids of revolution</li> <li>Disc method</li> <li>Washer method</li> </ul>
<b>Unit 7 - Review for Exam</b> Term 3 and Term 4 March through May	<ul> <li>Review for AP exam</li> </ul>



### **Content Standards**

- Demonstrate knowledge of both the formal definition and the graphical interpretation of limit of values of functions. This knowledge includes one-sided limits, infinite limits, and limits at infinity. Students know the definition of convergence and divergence of a function as the domain variable approaches either a number or infinity:
  - Prove and use theorems evaluating the limits of sums, products, quotients, and composition of functions.
  - Use graphical calculators to verify and estimate limits.
  - Prove and use special limits, such as the limits of  $\frac{(\sin(x))}{x}$  and  $\frac{1-\cos(x)}{x}$  as x tends to 0.
- Demonstrate knowledge of both the formal definition and the graphical interpretation of continuity of a function.
- Demonstrate an understanding and the application of the intermediate value theorem and the extreme value theorem.
- Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:
  - Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.
  - Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change.
     Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
  - Understand the relation between differentiability and continuity. 4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
  - Know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.
  - Find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.
  - Compute derivatives of higher orders.
  - Know and can apply Rolle's Theorem, the mean value theorem, and L'Hôpital's rule.
  - Use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
  - Know Newton's method for approximating the zeros of a function.
  - Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.
  - Use differentiation to solve related rate problems in a variety of pure and applied contexts.
  - Know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
  - Apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.
  - Demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.
  - Use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.
  - Compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.



### **Content Standards (continued)**

- Know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.
- Compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.
- Compute the integrals of trigonometric functions by using the techniques noted above. 21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.
- Understand improper integrals as limits of definite integrals.



### **Content Standards (continued)**

- Demonstrate an understanding of the formal definition of the derivative of a function at a point and the notion of differentiability:
  - Demonstrate an understanding of the derivative of a function as the slope of the tangent line to the graph of the function.
  - Demonstrate an understanding of the interpretation of the derivative as an instantaneous rate of change.
     Students can use derivatives to solve a variety of problems from physics, chemistry, economics, and so forth that involve the rate of change of a function.
  - Understand the relation between differentiability and continuity. 4.4 Students derive derivative formulas and use them to find the derivatives of algebraic, trigonometric, inverse trigonometric, exponential, and logarithmic functions.
- Know the chain rule and its proof and applications to the calculation of the derivative of a variety of composite functions.
- Find the derivatives of parametrically defined functions and use implicit differentiation in a wide variety of problems in physics, chemistry, economics, and so forth.
- Compute derivatives of higher orders.
- Know and can apply Rolle's Theorem, the mean value theorem, and L'Hôpital's rule.
- Use differentiation to sketch, by hand, graphs of functions. They can identify maxima, minima, inflection points, and intervals in which the function is increasing and decreasing.
- Know Newton's method for approximating the zeros of a function.
- Use differentiation to solve optimization (maximum-minimum problems) in a variety of pure and applied contexts.
- Use differentiation to solve related rate problems in a variety of pure and applied contexts.
- Know the definition of the definite integral by using Riemann sums. They use this definition to approximate integrals.
- Apply the definition of the integral to model problems in physics, economics, and so forth, obtaining results in terms of integrals.
- Demonstrate knowledge and proof of the fundamental theorem of calculus and use it to interpret integrals as antiderivatives.
- Use definite integrals in problems involving area, velocity, acceleration, volume of a solid, area of a surface of revolution, length of a curve, and work.
- Compute, by hand, the integrals of a wide variety of functions by using techniques of integration, such as substitution, integration by parts, and trigonometric substitution. They can also combine these techniques when appropriate.
- Know the definitions and properties of inverse trigonometric functions and the expression of these functions as indefinite integrals.
- Compute, by hand, the integrals of rational functions by combining the techniques in standard 17.0 with the algebraic techniques of partial fractions and completing the square.
- Compute the integrals of trigonometric functions by using the techniques noted above. 21.0 Students understand the algorithms involved in Simpson's rule and Newton's method. They use calculators or computers or both to approximate integrals numerically.
- Understand improper integrals as limits of definite integrals.